Technical Note

INSIGHT.DATA.CLARITY.

February 2018

THE CONVEXITY OF TREND FOLLOWING

Protecting your assets but perhaps not as much as you would like!

Executive Summary

Much has been made recently of crisis alpha or crisis risk offset. And, in particular, of using trend following as a hedge of future downside moves in, mostly, equity markets. We demonstrate that trend following is mechanically convex relative to the underlying upon which one is trending, but, that the overall convexity offered by CTAs is mitigated by implementation steps that improve risk and execution cost adjusted returns. Trend Following should primarily be viewed as a highly statistically significant strategy, while the existence of convexity, albeit weak, should be considered a bonus feature to an investment in Trend Following.

Contact details

Call us +33 1 49 49 59 49

Email us cfm@cfm.fr

Introduction

Commodity Trading Advisors (CTAs) have traded futures markets for many years using, on aggregate, an approach exploiting the persistence of trends in prices[1]. It has been demonstrated by many, the current authors included, that following trends is a profitable strategy with a high level of statistical significance[2]. The Sharpe ratios observed by a systematic approach are modest, as are the returns of the industry. These modest Sharpe ratios are, however, persistent – leading to the conclusion that the approach represents one of a growing number of alternative benchmark strategies upon which the alternative beta industry has been created.

However, traditional benchmarks still reign supreme, with equity indices composed of developed markets and/or emerging markets mixed in with both sovereign and corporate fixed income type return streams representing the backbone of most institutional investment portfolios. These investments are sound, with expected Sharpe ratios in the range of 0.3-0.4 for equities and, to first order, the excess yield of developed market sovereign bonds leading to low Sharpe ratios in low interest rate regimes[3]. The advent of the Alternative Beta industry has spurned interest in an alternative, decorrelated (and hopefully positive) return stream that can potentially help to decrease the volatility and drawdowns of investor portfolios.

Trend following, in particular, is an alternative benchmark strategy that exhibits certain features that help traditional portfolios. This was most strikingly illustrated through the Global Financial Crisis of 2008 when most traditional and alternative strategies sold off. The CTA industry, which, frankly speaking, until 2008 only occupied a sleepy corner of the Hedge Fund world, revealed itself to be one of the strategies that performed best during stressed markets. The SG CTA index was up 13.1% through the tumultuous financial crisis period of 2008 while many CTA managers recorded multiple returns of 3-4 standard deviations (or annualised volatility) for 2008. This obviously attracted a lot of investor attention and the CTA industry picked up the pieces from the crisis to become a prominent sector of the Hedge Fund universe.

The returns that followed the crisis from 2009-2013 disappointed many. Having invested on the back of reasonable returns over several economic cycles and spectacular performance in 2008, clients began to deliberate on the death of the trend in an overcrowded market, leading, ultimately, to a modest pullback from the industry globally. That this effect predated the dramatic 2014 draw-up in CTA performance is testament to investors' preponderance for following trends. All of a sudden CTAs were back in vogue, and investors and analysts began to give the industry a second look – in essence following the trend of trend following performance.

We begin our discussion of trend convexity by setting the scene with a mathematical framework. The less mathematically minded reader is encouraged to skip this section and move onto a discussion of our key equation that shows that trend following is mechanically convex relative to the instrument being trended upon. There are various additional techniques involved in building a CTA, however, and these can reduce the convex nature of the trend following strategy. Techniques such as: (i) trending on a diversified pool of instruments; (ii) imposing a maximum (a *cap*) on the trend forecast and, finally, (iii) slowing the strategy down such that it reacts only gradually to sudden underlying moves. We finish our discussion with a convexity study of the Société Générale (SG) CTA[4] index that shows how much protection the CTA industry really provides. We conclude with a summary of our results.

The mathematical framework

In mathematical finance, convexity refers to non-linearities in a financial model. In other words, if the price of an underlying variable changes, the price of an output does not change linearly, but depends on the second derivative (or, loosely speaking higher order terms) of the modelling function. Geometrically, the model is no longer flat but curved, and the degree of curvature is called the convexity.[5]

The term convexity is usually used in the context of options to refer to a payoff that depends non-linearly on price. A delta hedged at-the-money call or put option provides a large payout in the case of a large move of the underlying in either direction. This hedging property of options is frequently exploited – a holder of equities, for example, may choose to buy puts before a big market announcement to protect against potential large downside moves. A similarly shaped payoff for trend following has been known to exist for many years. The payoff for trend following is slightly different in that a

[1] See our Whitepaper "Explaining hedge fund index returns" in the 2017 Q4 Alternative Beta Matters Newsletter, available on the CFM website.

[2] See our academic paper *Two Centuries of Trend Following*. Y. Lempérière, C. Deremble, P. Seager, M.Potters, J.P. Bouchaud. 2014, Journal of Investment Strategies 3(3), 41-61.

[3] See our Whitepaper "Modelling Forward looking Returns and Combining Traditional and Alternative Benchmarks" in the 2016 Q3 Alternative Beta Matters Newsletter, available on the CFM website.

[4] The SG CTA Index is widely recognised as the key benchmark for managed futures and is calculated by Société Générale. See https://cib.societegenerale.com/en/prime-services-indices/

[5] Definition taken from https://en.m.wikipedia.org/Convexity_(finance)

downside (or upside) move needs to persist over a timescale comparable to the trend timescale, but in such situations trend following will pay out.

We can quickly write a mathematical prescription for this effect. For the purposes of illustration we take a simplified trend following approach with a forecast *F(t)* at time *t* defined relative to an arbitrary time t_0 in the past.

$$F(t) = p(t) - p(t_0)$$

Where *p(t)* is the price of the instrument at time *t* . We also define a daily price return $\delta p(t)$ at time *t* as follows:

$$\delta p(t) = p(t+1) - p(t)$$

Assuming the trend follower takes on a position equal to *F(t)* then the Profit and Loss (P&L) for any one day is equal to:

$$P\&L(t) = F(t)\delta p(t)$$

One can now sum over all days from t_0 to the final day, *T*, to produce the total P&L as the following:

$$P\&L_{total} = \sum_{t_0}^{t_0+T} (\delta(t) - \delta(t_0))\delta p(t)$$

This can then be rearranged to produce the following equality:

$$P\&L_{total} = 1/2\left[\left(\sum \delta p(t)\right)^2 - \sum (\delta p(t))^2\right]$$

This equation shows that the P&L of a trend follower depends on the difference of the variance[6] of the timeseries on two timescales. One can calculate the variance on the timescale of days, weeks, months, decades etc. albeit with less and less precision for any one given timeseries. For a *random walk*[7] these variances are related by a factor proportional to the difference in time i.e:

$$\sigma_{1\,day}^2 = 5\sigma_{1\,week}^2 = 20\sigma_{1\,month}^2 = etc.$$

These equalities change, however, in the presence of positive and negative autocorrelation in the timeseries. Looking at the random walk timeseries plotted in Figure 1 we see that a trend following strategy performs well when the variance defined on the timescale, T, close to that of the trend, is higher than T times that of the variance measured on a daily timescale. Conversely, of course, when the opposite is the case the trend strategy performs poorly. This result is very intuitive – trend following is a good strategy when there is a move, up or down, that persists for a timescale comparable to the trend timescale.

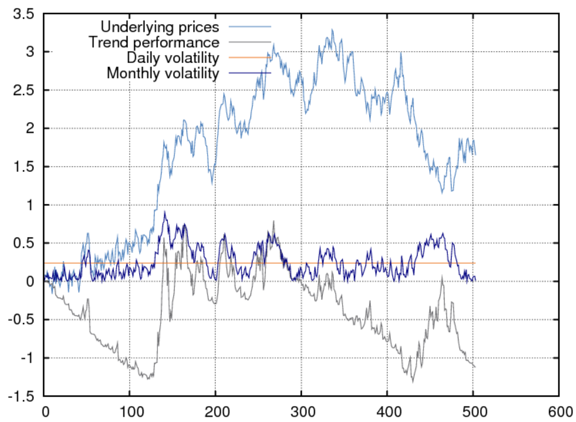

Figure 1 – The simulated evolution of the prices of a random walk with the result of a trend following strategy overlaid as a function of time (the number of days in this case). The random walk is by definition unpredictable and therefore the long term performance of the trend is strictly zero. However, at times the strategy can perform well and at other times will underperform. The above derivation tells us that in times of the variance measured on the timescale of the trend being greater than T times that measured on the timescale of a day the trend will perform positively and negatively otherwise. One also notes that the trend strategy loses more often than it gains, and, when it gains, it gains more than it loses. In other words, the trend strategy is positively skewed[8] on the timescale of the trend.

A more standard way to follow trends is to use an Exponentially weighted Moving Average with a decay rate on a timescale τ. The forecast *F(t)* is then:

$$F(t) = EMA_\tau^{t-1}(\delta p(t))$$

Where the EMA is defined as:

$$EMA_\tau^{t-1}(X(t)) = \sum_{t' \le t-1} e^{-t'/\tau} X(t')$$

For any variable X. In such a case one can show that[9]:

$$EMA_{\tau/2}(P\&L(t)) \propto \tau[EMA_\tau(\delta p(t))]^2 - EMA_{\tau/2}(\delta p(t)^2)$$

Equation 1

Where the P&L is now defined as:

$$P\&L(t) = F(t)\delta p(t)$$

[6] Variance is the square of the standard deviation, or volatility. Volatility is defined as the degree of variation in the price time series of a security as measured by its standard deviation. A daily volatility is mathematically defined as $\sigma = 1/T\sqrt{\sum_{t=1}^{T} \delta p_t}$ where δp is the daily return.

[7] A Random Walk, for our purposes, follows a price trajectory given by $p(t) = \sum_{t'=0}^{t} \eta_{t'}$ where η corresponds to a series of bell shape, distributed random numbers. These random numbers are a representation of the price returns of a financial instrument. The timeseries generated in this way is unpredictable and has a Sharpe ratio of zero.

[8] This is addressed in a subsequent white paper in preparation

[9] Please see our academic paper "Tail protection for long investors: Trend convexity at work". Available at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2777657 for a derivation

Writing the dependence of P&L against the performance of the underlying timeseries in this way enables us to clearly see the convexity. We will see this visually in the below.

Trend following is mechanically convex!

Welcome back if you have skipped the math section! That being the case we ask, nonetheless, that you refer to *Equation 1* above which puts us in a position to investigate the convex behaviour of the trend. Think back to high school math, plotting out a function $y = x^2$ gives a U-shaped *parabola*. Similarly plotting out an Exponentially weighted Moving Average (EMA) of the P&L of trend following on an instrument against the EMA of the returns of the instrument itself also gives us this same U shaped form (*i.e.* we are plotting $y = x^2$). The relative timescales used in the EMAs have to be correct (see *Equation 1*) but that being the case the equality is exact, even for a random walk that is by definition unpredictable. The third term in *Equation 1* is the daily volatility of the instrument upon which we are trending. For a random walk this is constant, again, by definition[10]. For financial instruments this term, the volatility, varies through time but risk control techniques can be used to keep this constant by buying and selling amounts inversely proportional to a forecast of volatility[11]. This aspect of risk control is essential to being a successful CTA!

As demonstrated above, it is a feature of trend following that persistent and cumulatively large moves in the underlying produce big returns for the strategy. *Figure 2* uses *Equation 1* to demonstrate this, first, by using a zero Sharpe ratio random walk. We plot an EMA of trend performance against an EMA of price returns and observe the characteristic convex U-shape. Even for a random walk, periods of big, cumulative moves in the underlying timeseries produce good performance for trend following (any strategy can correctly forecast the market for a statistically insignificant period of time!) The random walk timeseries is unpredictable and the average over all points on the y-axis is indeed zero – one cannot make money in the long term by trending on a random walk – but the performance can be more or less positive depending on the recent trajectory of the price. *Figure 2* also shows this convexity feature using S&P 500 and Wheat futures. In the case of the S&P 500, *Equation 1* is satisfied showing convexity of trend following relative to big, cumulative moves in the S&P 500. Similarly, trend following on Wheat is convex relative to moves in the Wheat market. Trend following therefore provides protection against big, cumulative moves of anything! What people are often concerned about is protecting their portfolio of stocks! We therefore focus our attention on the particular case of the convexity with respect to the S&P 500 *only* when building a diversified trend portfolio.

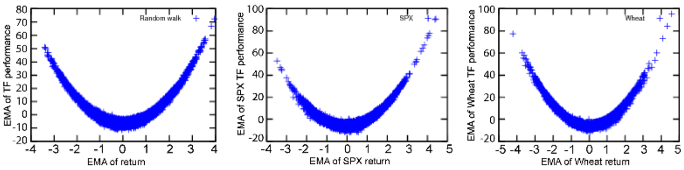

*Figure 2 – Equation 1 is used to produce scatter plots of three separate timeseries – a random walk (left), the S&P 500 (middle) and Wheat (right). In each case the timeseries is approximately 35 years long. The y-axis is the left hand side of **Equation 1** – an EMA over a timescale $\tau/2$ of the P&L of a trend following strategy using an EMA on a timescale τ. The x-axis is the EMA of price returns on a timescale τ. The convexity is observed (a U-shaped parabola) for all values of τ. In the plots we use a characteristic timescale of a couple of months. In each case the P&L is normalised to have a daily volatility of 1%.*

How convex are trend followers really?

In this section we would like to consider the impact of various implementation techniques that trend followers typically employ, all of which serve to improve risk adjusted returns, but, at the detriment of the convexity features discussed. The first step is to consider that a typical CTA trades a universe of instruments. It takes a moment's reflection, given what we know so far, to realise that adding in other instruments will reduce the convexity to any one single underlying. Investors are often looking for protection against downside moves in equity markets. Let us consider a scenario where we have a universe of 9 decorrelated instruments as seen in *Figure 3*. We build up the universe steadily from the first – the S&P 500 – before adding in a trend strategy on the second, and the third etc. all the while plotting out the same EMA of performance relative to the EMA of the S&P 500. As we add in more and more contracts, one observes a blurring of the convexity picture. This is normal! Trending on each instrument is convex with respect to the instrument but not with respect to the S&P 500. One does notice however that the performance is enveloped by a parabola from below, which is simply a consequence of adding the original true parabola to a mass of points that are independent of the first random walk. It is worth noting, also, that a lot of instruments in the CTA universe are correlated, in particular during periods of market stress, and therefore some convexity relative to the stock market

[10] Generally the standard deviation of the random number, in our case η, is 1

[11] We have a white paper in preparation that explains risk control in more detail

remains. Take for example a portfolio that follows trends on developed market government bond and equity index futures. In a crisis scenario the equity index futures collapse and the bonds rise. The trend strategy makes money on both, and convexity, relative to the stock market is maintained. However, a typical CTA portfolio is full of many other types of instruments that ultimately reduce the level of equity convexity.

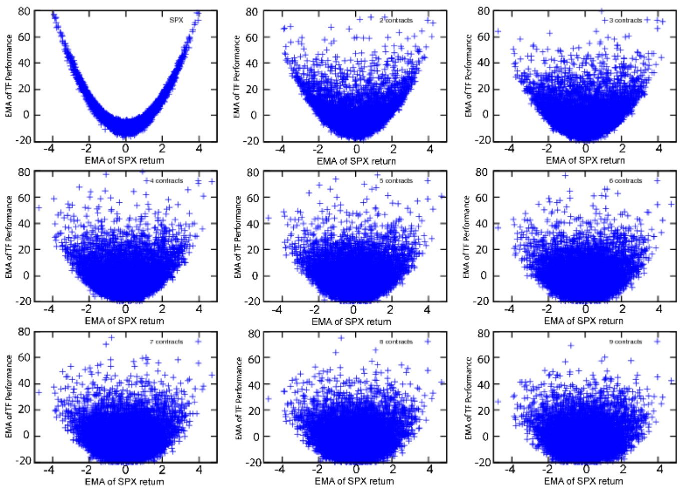

Figure 3 – The convexity of a fictitious portfolio of instruments. The SPX is actually a random walk and each subsequent plot represents the addition of another independent instrument (more random walks!) to the portfolio. We plot the convexity of the portfolio with respect to moves in the first plot in order to demonstrate the reduction of convexity in a diversified CTA portfolio for those seeking protection against moves in the equity markets. One notices, however, that each is bounded/enveloped from below by the original parabola, implying of course that some convexity is maintained.

We next move on to a real world example taking two contracts – the S&P 500 and Wheat – and we plot out the convexity of the combination with respect to moves in the S&P 500. These two instruments have been chosen since they are decorrelated (measured correlation is close to zero) and available over a long history. In Figure 4 one observes the convexity of the combined S&P 500/Wheat portfolio relative to the S&P 500 to be less clear than in the case of trend following on the S&P 500 alone. Of course the Sharpe ratio of trend following on a diversified portfolio of instruments is higher than on any one on a standalone basis. It is for this reason that CTAs trade a pool of instruments that is as diversified as possible. This comes, however, at the expense of convexity with respect to any one contract including, therefore, equity indices. Wheat is however just as well hedged against big moves in the Wheat market as the S&P 500 is against big equity moves. Investors are generally not so concerned by this though! It remains the case, however, that if one wants to hedge exposure to big, cumulative moves in any one instrument it is better to build a trend following strategy on that instrument. Diversifying reduces that convexity *but* improves the Sharpe ratio.

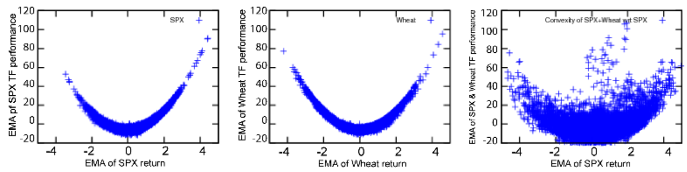

Figure 4 – The convexity of trend following on real data. The first plot is the S&P 500 while the second is Wheat futures. The final plot (far right) is the convexity of the combined trend following P&L of S&P 500 and Wheat relative to moves in the S&P 500 alone.

Building a diversified portfolio is not the only strategy that CTAs have to make trend following returns more palatable. The convexity defined so far assumes that as a trend becomes bigger then so does a manager's position in that instrument *i.e.* the position is proportional to the trend. At the time of writing of this note (beginning of 2018) equity indices have been on a prolonged and increasingly steep rally. It seems unreasonable to continue increasing one's exposure to such a trend, and indeed, most managers *cap* their positions when the trend reaches a given level - which is generally early in the formation of a trend. One can take this to an extreme to illustrate the effect on convexity: instead of looking at the performance of a trend follower where the position is proportional to the size of trend, on the y-axis we plot an EMA of the *performance of the sign of the trend predictor*: if positive we are long 1 unit and if negative we go short 1 unit. This is seen in Figure 5 which shows, once more, a reduction in convexity. The effect on the P&L of this step is to reduce the size of the fat tails of the returns. If one continues to build up a position as trends get bigger, one assumes more risk infrequently, the definition of *Kurtosis*[12], or fat tails.

Also seen in Figure 5 is another step in the construction of the trend signal commonly employed by CTAs. Execution costs from trading in markets are high and investment managers are often on the lookout for ways of reducing the amount of trading they do. A common way to do this, for all strategies, is to average the predictor over recent observations. One can therefore do an EMA of the trend predictor, itself already an EMA of price returns. This extra layer of averaging slows the strategy significantly and successfully maintains the level of risk adjusted returns. However, such a change to the strategy affects the

[12] Defined according to Investopedia.com: "Kurtosis is a statistical measure that's used to describe the distribution, or skewness, of observed data around the mean, sometimes referred to as the volatility of volatility. Kurtosis is used generally in the statistical field to describe trends in charts. Kurtosis can be present in a chart with fat tails and a low, even distribution, as well as be present in a chart with skinny tails and a distribution concentrated toward the mean."

convexity. Imagine a move in the S&P 500 which results in a trade being generated by a trend follower. An extra EMA on top of the trend delays this trade, spreading it out over several days with a position that is subsequently entered into over a drawn out period depending on the timescale of the extra averaging. This is another typical CTA trick to increase the Sharpe ratio of the overall portfolio but at the expense of the convexity behaviour.

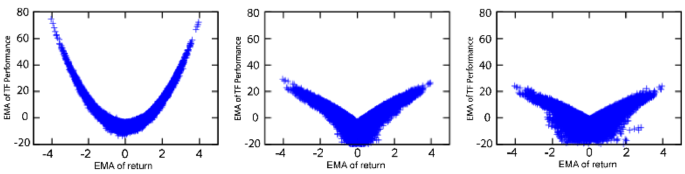

Figure 5 – The convexity of a random walk (left) compared to the convexity of a trend following strategy where position is set to +1 or -1 based on the sign of the trend predictor (middle). This is a common way to reduce the size of tails in the return distribution of a trend follower - a manager's fat tails can be very detrimental to a business! The final plot (right) is the same plot after having smoothed the predictor with another EMA that slows the trading to keep execution costs under control. Both techniques help to achieve better risk adjusted returns but are detrimental to convexity with respect to any one underlying instrument.

We are now in a position to study the convexity of the CTA industry as a whole. A previous white paper[13] which successfully explained the aggregate performance of the CTA industry using trend following alone is used in order to define the timescale over which the aggregate market trends. The study reproduced the index with a correlation of about 85% by trend following on futures contracts on equity indices, interest rates, commodities and FX. The timescale that explained best the performance of the SG CTA index, found to be of the order of 6 months, can now be used as an input to the convexity plot. In *Figure 6* we plot an EMA of the SG CTA index against the corresponding EMA of the S&P 500, with all timescales appropriately used according to *Equation 1*. Also shown on the plot is a $y = x^2$ line with a calculated R^2 of 0.18. The R^2 is a measure of how well the points fit the $y = x^2$ line:[14] the closer the R^2 is to 1 the better the points respect the line. Also shown is the more standard way to demonstrate CTA convexity where one plots the monthly returns of the SG CTA index against the monthly returns of S&P 500. Here the equivalent R^2 is much worse at 0.02 and the fit to a $y = x^2$ line is frankly much less convincing. It is clear that the prescription in *Equation 1* accurately captures the real convexity of the aggregate CTA industry as represented by the SG CTA index. The plot resembles and exhibits the features of the various portfolio construction steps defined above. The convexity relative to the S&P 500 is present but ultimately mitigated by the desire to deliver stable risk adjusted returns.

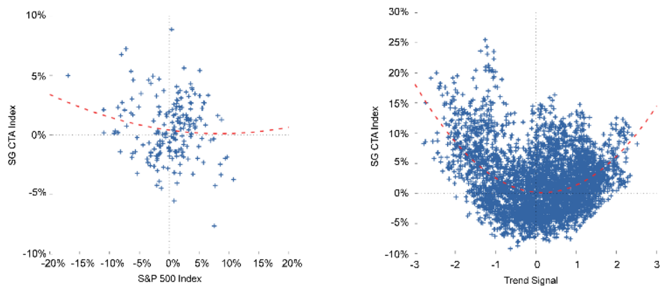

*Figure 6 – The standard way to look at CTA convexity (left) by plotting the monthly returns of the SG CTA index against monthly returns of the S&P 500. The resulting fit of $y = x^2$ is unconvincing with an R^2 of 0.02. The convexity plotted as described by **Equation 1**, however, has an R^2 of 0.18 using the best fitted trend timescale to describe the SG CTA index. Also of note is the enveloping parabolic shape from below that resembles the convexity demonstrated above in mixing a trend following approach on many instruments.*

Conclusions

There are various ways to achieve portfolio protection and trend following is commonly used in this way. Buying short dated debt of the most creditworthy government i.e. US T-bills is also a common strategy but the income or capital gain generated is only through changes in interest rates and the carry obtained through the slope of the yield curve. Alternatively one may choose to buy put options on an equity index. This is a systematically negative strategy, however, consistent with the risk premium nature of the P&L associated with buying insurance[15]. Trend following, on the other hand, has convex features to its P&L and is also robustly positive in its long term performance.

The authors of this note, along with many others, have shown that systematic trend following is statistically significant; robust across instruments and through time; robust to changes in parameters; in a slow form relatively insensitive to costs; and also plausible in that it likely exploits a behavioural bias. The strategy is mechanically convex relative to the instrument upon which one is trending. It is less convex to that same instrument upon the inclusion of other instruments in the portfolio and other typical changes to the strategy that make it viable for running a successful trading program and a successful business. Nonetheless, it is a robust strategy and it is our belief that it is better to invest in trend following for its

[13] Refer again to our Whitepaper "Explaining hedge fund index returns" in the 2017 Q4 Alternative Beta Matters Newsletter. See also our academic paper "Tail protection for long investors: Trend convexity at work". Available at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2777657

[14] See the definition and interpretation of R^2, or the coefficient of determination: https://en.wikipedia.org/wiki/Coefficient_of_determination

[15] See our academic paper *Risk premia: asymmetric tail risks and excess returns*. Y. Lempérière, C. Deremble, T.T. Nguyen, P. Seager, M.Potters, J.P. Bouchaud. 2017, Quantitative Finance 17(1), 1-14. The reader can also refer to our Whitepaper "Risk Premium Investing – A Tale of Two Tails" in the 2015 Q4 Alternative Beta Matters Newsletter available on the CFM website.

robustness than for the protection offered, although any protection is obviously a bonus!

End note

All references within to 'S&P 500' is intended as shorthand in referring to the S&P 500 Mini futures contract (Bloomberg ticker: ESA). Likewise, all reference to 'Wheat' is shorthand for the first generic futures contract (Bloomberg ticker: W 1)

Disclaimers

CFM has pioneered and applied an academic and scientific approach to financial markets, creating award winning strategies and a market leading investment management firm.

Contact us

Capital Fund Management S.A.
23, rue de l'Université, 75007
Paris, France
T +33 1 49 49 59 49
E cfm@cfm.fr

CFM International Inc.
The Chrysler Building, 405 Lexington Avenue - 55th Fl.,
New York, NY, 10174, U.S.A
T +1 646 957 8018
E cfm@cfm.fr

CFM Asia KK
9F Marunouchi Building, 2-4-1, Marunouchi, Chiyoda-Ku,
100-6309 Tokyo, Japan
T +81 3 5219 6180
E cfm@cfm.fr

Capital Fund Management LLP
64 St James's Street, London
SW1A 1NF, UK
T +44 207 659 9750
E cfm@cfm.fr